

*Все, что познается, имеет число,  
ибо невозможно ни понять ничего,  
ни познать без него.  
Пифагор*

# Математичні методи, моделі та інформаційні технології в економіці

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## **USING FUZZY COMPUTING FOR MAKING DECISIONS ON THE FORMATION OF AN INVESTMENT PORTFOLIO**

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The diversification of the investment portfolio may be regarded as one of the ways to manage investment risk. One of the solutions to this problem is the approach of Markowitz. However, it uses a number of assumptions which are poorly consistent with the realities of investment processes. Thus, the requirement of statistical homogeneity cannot be achieved in real conditions. The use of to the subjective probabilities almost does not improve the situation.

It is assumed that there are some projects (investment projects, food programs, securities) from which an investment portfolio is to be formed and investments in these projects should be appropriately distributed. The information about the projects is vague and its possible refinement is associated with unacceptable time and material costs. Besides, the necessary level of certainty is not guaranteed. The resulting estimates are expert ones and they do not always have a quantitative representation, often being approximate.

A mathematical substantiation, an algorithm and practical implementation of the solution to the problem are given, this problem being regarded as a fuzzy analogue of a statistical game. This problem is formulated in a fuzzy statement and several ways to solve it are presented.

An algorithm and computational and analytical methods of making a rational decision on the formation of the investment portfolio have been described. These methods are free from defects of other known approaches, making it possible to take into account the multiplicity of identical estimates of yield components of the investment portfolio which ultimately enhances the validity of the distribution of investment resources.

The presented approach has been successfully applied to practice in the assessment of the options and management and economic decision-making in the economic analysis and portfolio management in a number of commercial banks.

*Keywords:* investment portfolio formation, fuzzy modeling, investment decision-making.

## ЗАСТОСУВАННЯ НЕЧІТКИХ ОБЧИСЛЕНЬ ПІД ЧАС УХВАЛЕННЯ РІШЕНЬ ПРО ФОРМУВАННЯ ІНВЕСТИЦІЙНОГО ПОРТФЕЛЯ

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Диверсифікація інвестиційного портфеля може розглядатися як один із способів управління інвестиційними ризиками. Одним із варіантів вирішення цього завдання є підхід Марковіца. Однак він використовує ряд припущень, недостатньо узгоджуваних із реаліями інвестиційних процесів. Так, умова статистичної однорідності не може бути забезпечена в реальних умовах. Звернення до суб'єктивних імовірностей також практично не поліпшує ситуацію.

Передбачено, що існує деяка кількість проектів (інвестиційні проекти, продуктові програми, цінні папери), із яких необхідно побудувати інвестиційний портфель і розподілити відповідним чином вкладення в ці проекти. Відомості про проекти мають розпливчастий характер і можливості їх уточнення пов'язані з неприйнятними тимчасовими й матеріальними витратами, також не гарантовано досягнення необхідного рівня визначеності. Здобуті оцінки є експертними, не завжди мають кількісний вираз і часто є приблизними.

Наведено математичне обґрунтування, алгоритм і практична реалізація вирішення поставленого завдання, котре розглядають як нечіткий аналог статистичної гри. Це завдання сформульовано в нечіткій формі й надано кілька способів його вирішення.

Запропоновано алгоритм і розрахунково-аналітичну методичку ухвалення раціональних рішень щодо формування інвестиційного портфеля, вільну від недоліків інших відомих підходів, що дає можливість урахувати кратність однакових оцінок прибутковості компонент інвестиційного портфеля. У кінцевому підсумку це дозволяє підвищити обґрунтованість розподілу інвестиційних ресурсів.

Наведений підхід успішно застосовувався на практиці у процесі оцінювання варіантів та ухвалення управлінсько-економічних рішень під час аналізу та управління інвестиційними портфелями в ряді комерційних банків.

*Ключові слова:* формування інвестиційного портфеля, нечітке моделювання, ухвалення інвестиційних рішень.

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## ПРИМЕНЕНИЕ НЕЧЕТКИХ ВЫЧИСЛЕНИЙ ПРИ ПРИНЯТИИ РЕШЕНИЙ О ФОРМИРОВАНИИ ИНВЕСТИЦИОННОГО ПОРТФЕЛЯ

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Диверсификация инвестиционного портфеля может рассматриваться как один из способов управления инвестиционными рисками. Одним из вариантов решения этой задачи является подход Марковица. Однако он использует ряд предположений, недостаточно согласуемых с реалиями инвестиционных процессов. Так, условие статистической однородности

не может быть обеспечено в реальных условиях. Обращение к субъективным вероятностям также практически не улучшает ситуацию.

Предположено, что существует некоторое количество проектов (инвестиционные проекты, продуктовые программы, ценные бумаги), из которых необходимо построить инвестиционный портфель и распределить соответствующим образом вложения в эти проекты. Сведения о проектах имеют расплывчатый характер и возможности их уточнения связаны с неприемлемыми временными и материальными затратами и не гарантировано достижение необходимого уровня определенности. Получаемые оценки являются экспертными, не всегда имеют количественное выражение, часто носят приблизительный характер.

Приведено математическое обоснование, алгоритм и практическая реализация решения поставленной задачи, рассматриваемой как нечеткий аналог статистической игры. Данная задача сформулирована в нечеткой форме и представлено несколько способов ее решения.

Предложен алгоритм и расчетно-аналитическая методика принятия рационального решения по формированию инвестиционного портфеля, свободная от недостатков других известных подходов, дающая возможность учесть кратность одинаковых оценок доходности компонент инвестиционного портфеля. В конечном итоге это позволяет повысить обоснованность распределения инвестиционных ресурсов.

Представленный подход успешно применялся на практике при оценке вариантов и принятии управленческо-экономических решений при анализе и управлении инвестиционными портфелями в ряде коммерческих банков.

*Ключевые слова:* формирование инвестиционного портфеля, нечеткое моделирование, принятие инвестиционных решений.

As is well known, the diversification of the investment portfolio can be viewed as one of the most important ways to manage investment risks [1].

One of the possible solutions to this problem is the Markowitz approach. Although it was widely used in the practice of portfolio management, however, this approach uses a number of assumptions which badly correspond with the realities of investment processes [2; 3].

It is obvious, that the statistical homogeneity condition cannot be practically achieved in the real economy. Any economic entity or process studied and analyzed in the past time, and the same object at the moment are, in general, two different objects of observation and investigation. It is explained by the fact that the market environment of a project changes over time. Accordingly its market position also changes.

So the risk of losses in a specific business sector will fall or rise, but the cause of these fluctuations will actually be in the external business environment. All these facts don't allow anyone to speak about statistical homogeneity of the studied process.

An appeal to subjective probabilities practically does not improve the situation.

If a probabilistic subjective assessment is made by only one expert, then the risk of subjectivity and an erroneous forecast significantly increase. In fact, when an expert applies subjective probabilities, he refuses from the frequency approach and puts his subjective expectations in the concept of probability, which are largely based on the history and on the past experience. At the same time due to the fact that market conditions are unplayable, this prehistory ceases to be indicative and reliable for further calculations and forecasting.

It is well known that the usage of correlation coefficients suggests the immutability for the character of causal

relationships. However, in actual economical practice this does not take place. Firstly, both the character and the relations themselves at the time of the study may be not fully defined. Secondly, there is a possibility of significant changes under the action of external forces and factors.

At the same time there have been successful researches on the application of fuzzy sets to the construction of investment portfolios.

The authors propose a solution to this problem, which can be considered as a fuzzy analogue of a statistical game [4].

For a start the authors assume that there are a lot of various projects (investment projects, food programs, securities, etc.) from which one has to build an investment portfolio. In other words it is necessary to properly distribute the investments in these projects.

The authors also assume that the information about the projects is vague, indefinite, and at this stage of investigations its refinement is time-consuming and costly. And the present situation cannot guarantee the achievement of the desired (required) level of certainty.

The resulting estimates can be classified as the expert ones and it is not always possible to view them in a quantitative form. In addition, quantitative estimates are rather approximate. This problem set in fuzzy formulation can be solved in several ways [5 – 10]. At first the authors propose to consider the simplest way.

Suppose, there are several projects to form an investment portfolio  $S = \{s_j : j = \overline{1, N}\}$ . There is also an assumption that estimates are known for possible incomes in the implementation of these projects:

$$\tilde{C} = \left\| \tilde{C}_{i,j} \right\|. \quad (1)$$

Suppose also that a set of possible combinations of these yields has been constructed and expert assessments of the probability of these combinations have been identified:  $\tilde{p}_i$ ,  $i = \overline{1, M}$ , where  $M$  is the number of considered combinations of the portfolio component yields which form a set of  $P$ :

$$\tilde{P} = \{ \tilde{p}_i; i = \overline{1, I_p} \}. \quad (2)$$

In other words, for the relationship  $\tilde{P}/S \ S_1 \ \dots S_n$  there is a matrix of probability estimates:

$$\begin{matrix} \tilde{P}_1 \\ \tilde{P}_2 \\ \tilde{P}_3 \\ \dots \\ \tilde{P}_m \end{matrix} \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{1n} \\ \tilde{C}_{21} & \tilde{C}_{2n} \\ \tilde{C}_{31} & \tilde{C}_{3n} \\ \dots & \dots \\ \tilde{C}_m & \tilde{C}_{mn} \end{pmatrix}. \quad (3)$$

The sets (1) and (2) can be associated with linguistic assessment of their possible values. Thus, for the possibilities of the implementation of  $P$  one can consider  $L_p = (l_{p_i} : i = \overline{1, I_p})$ ,  $M_p = \{ \mu_{l_{p_i}}(z) / z \}$ , and respectively, for yield  $C$  one can consider:  $L_c = \{ l_{c_j} : j = \overline{1, J_c} \}$  and  $M_c = \{ \mu_{l_{c_j}}(y) / y : j = \overline{1, J_c} \}$ .

For simplicity, it is assumed that  $z \in [0,1]$  and  $y \in [0,1]$ . For variable  $z$  this condition is quite natural, since it is a matter of probabilities. For variable  $y$  its belonging to the interval  $[0,1]$  indicates the need to pass from the absolute scale to a relative one. For simplify of presentation it is assumed that the set  $L_p$  consists of 3 elements:

$L_p = \langle \text{small, medium, large} \rangle = \langle S, M, B \rangle$  (as shown in Fig. 1, on the left), and correspondingly  $L_c$  consists of 5 elements:

$L_c = \langle \text{small, less than medium, medium, more than medium, large} \rangle = \langle S, LM, M, MM, B \rangle$  (as shown in Fig. 1, on the right).

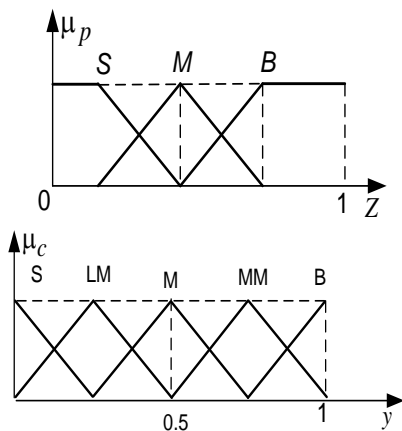


Figure 1. Membership functions for linguistic probability estimates (left) and yield (right)

Triangular membership functions (Fig. 1) were chosen for reasons of simplicity and only for elementary graphical representation. Estimates presented in Fig. 1 can be

constructed in various ways. Let us consider some of them with the assessment of yield taken as an example.

The first variant is that the investor's experts indicate the boundaries of a possible yield  $[Y_{\min}, Y_{\max}]$ . Then, within this interval we can define subintervals, which, according to the experts' opinions can match the selected linguistic estimates, such as those indicated in Fig. 1.

If computer software and a corresponding information system have been developed for the solution of the problem, then interactively appropriate membership functions can be constructed on the respective subintervals based on the established rules. They can be defined either by an expert, or selected from a set proposed by the mentioned software system. After that, it is converted to a relative scale.

Another variant may be illustrated by Fig. 2.

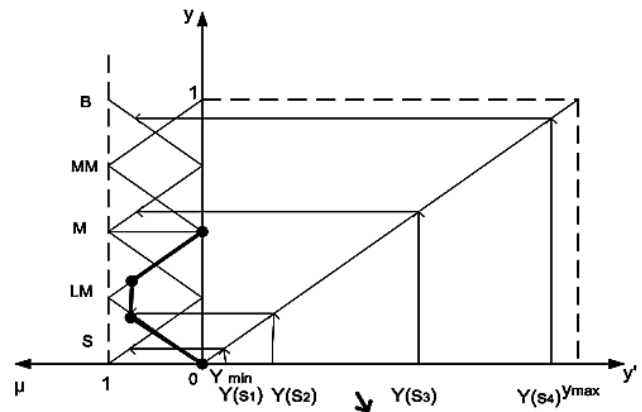


Figure 2. The graphical method of transition from the numerical estimates of yield for components of the investment portfolio to linguistic assessments

In this figure  $Y(s_i)$  is the numerical assessment of yield of the  $i$ -th component of the investment portfolio,  $i = 1, 2, 3, 4$ ;  $Y(s_1) \rightarrow S$ ,  $Y(s_2) \rightarrow LM$ ,  $Y(s_3) \rightarrow M$ ,  $Y(s_4) \rightarrow B$  is the linguistic evaluation of components of the portfolio (the thick lines show the type of the membership function for  $Y(s_2) \rightarrow LM$ ).

In the general case the conversion function can be nonlinear as well. As a result, the matrix (3) will contain only linguistic evaluations for a four-component portfolio. It may be, for example, as shown in Table 1.

Table 1

The numerical example  
of a utility matrix for a four-component of portfolio  
(the calculations are done for a special environment  
for operations with fuzzy numbers  
and variables named Fuzicalc)

	A	B	C	D	E	F	G
53		probability	assessments of profitability for components of the investment portfolio				
54		P/S	S1	S2	S3	S4	
55							
56		S	*	B	MM	B	B
57		S	*	B	B	MM	MM
58	$w=$	M	*	M	MM	MM	M
59		M	*	M	S	M	S
60		B	*	S	S	M	M
61		B	*	S	M	M	S
62							



The disadvantage of the proposed method is the use of the operation of intersection in the construction of fuzzy sets  $\tilde{w}_j$ .

It is a quite possible situation, when some  $\tilde{w}_{ij} = \emptyset$ , or in an extreme case for some  $j$ -th component of the portfolio all  $\tilde{w}_{ij} = \emptyset$ . Nevertheless, this result does not mean that the corresponding component of the portfolio should unequivocally be deleted from further consideration.

To resolve this situation, instead of the operations of intersection one can use another operation defined as the shadow of a fuzzy set  $Sh(\tilde{A}, \tilde{B})$ .

The shadow of the fuzzy set  $\tilde{A}$  to the fuzzy set  $\tilde{B}$  must satisfy the following requirements:  $Sh(\tilde{A}, \tilde{B})$  is the fuzzy set;  $Sh(\tilde{A}, \tilde{A}) = \tilde{A}$ ;  $Sh(\tilde{A}, \tilde{A}) = \emptyset$ , if at least one of the sets  $\tilde{A}$  or  $\tilde{B}$  is empty or the sets  $\tilde{A}$  and  $\tilde{B}$  are orthogonal.

The procedure for constructing the shadow of the fuzzy set  $\tilde{A}$  to the fuzzy set  $\tilde{B}$  can be defined as follows (Fig. 4):

$$Sh_{\varphi}(\tilde{A}, \tilde{B}) = \{\varphi[\mu_{\tilde{A}}(y), \mu_{\tilde{B}}(x)]/[y, x' = f(y)]\},$$

where  $f(y) = \frac{CG[\mu_{\tilde{B}}(x)]}{CG[\mu_{\tilde{A}}(y)]}y$  is the projection function;

$CG[\mu_{\tilde{B}}(x)]$ ,  $CG[\mu_{\tilde{A}}(y)]$  are the coordinates of the centers of gravity of the figures which are bounded by the membership functions  $\mu_{\tilde{A}}(y)$  and  $\mu_{\tilde{B}}(x)$ ;

$\varphi$  is the functional which sets the way for the transformation of the membership functions.

Fig. 5 shows the membership functions of fuzzy estimates of the probability and the components of the investment portfolio, which were used in the further calculations.

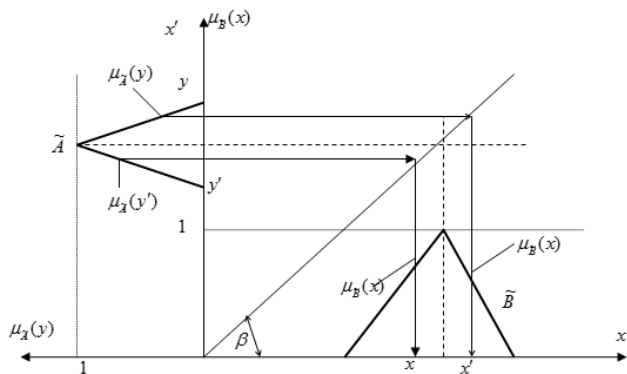


Figure 4. The geometric representation of the operation "the fuzzy set shadow"

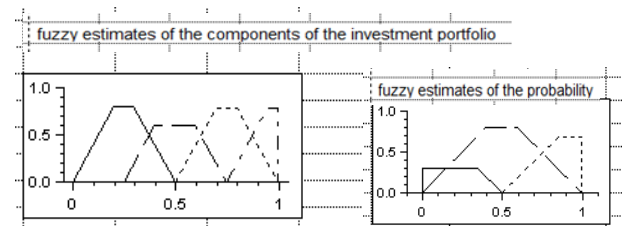


Figure 5. Membership functions of fuzzy estimates of the probability and the components of the investment portfolio

The sequence of computations used in the operation "the fuzzy set shadow" is as follows. For each column  $S_j$ ,  $j = \overline{1, N}$  one calculates:

$$\tilde{w}_{i,j} = Sh(\tilde{p}_i, \tilde{c}_{i,j}) = \{\varphi[\mu_{\tilde{p}_i}(y), \mu_{\tilde{c}_{i,j}}(x)]/[y, x' = f(y)]\}, \quad (9)$$

where  $i = \overline{1, 2, \dots, m}$ ,  $i \in [1, m]$ ;

$$f(y) = \frac{CG[\mu_{\tilde{c}_{i,j}}(x)]}{CG[\mu_{\tilde{p}_i}(y)]}y$$
 is the projection function,

$x \in [0, 1]$ ,  $y \in [0, 1]$ ;

$CG(\mu_{\tilde{c}_{i,j}}(x))$ ,  $CG(\mu_{\tilde{p}_i}(y))$  are the coordinates of the centers of gravity of the figures which are bounded by the membership functions  $\mu_{\tilde{p}_i}(y)$ ,  $\mu_{\tilde{c}_{i,j}}(x)$ ;

$\varphi = \min$  is the used functional which gives specifying transformations over the membership functions.

Then  $\tilde{w}_j = \bigcup_i \tilde{w}_{ij} = \max_i[\tilde{w}_{ij}]$  is found.

The results of the transformations according to Table 1 and Fig. 5 with the use of the relations (9) are shown in Fig. 6.

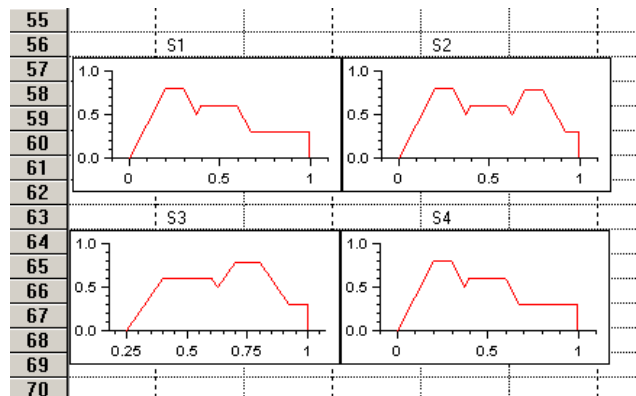


Figure 6. The membership functions of the resulting estimates for components of the investment portfolio

Further calculations are carried out according to equations (6 – 8) in the program tools Fuzicalc, and their results are given in Table 3.

Table 3

The calculation results for the matrix of linguistic evaluations (calculations have been made in the Fuzicalc environment)

coefficients of proportionality for components of the investment portfolio			
S1	S2	S3	S4
0.221205;	0.246747;	0.310843;	0.221205;
coefficients of multiplicity of ratings			
0.8;	1.5;	2;	1;
coefficients of proportionality for components of the investment portfolio with multiplicities of ratings			
0.127556;	0.266205;	0.44714;	0.159099;
(0.13);	(0.26);	(0.45);	(0.16);

The comparison of the results given in Tables 2 and 3 shows their obvious qualitative coincidence.

The difference in the numerical values can be explained by the fact that the use of the operation  $Sh$  provides for

a fuller taking into account all the aspects of a decision-making situation.

The proposed method of the investment portfolio formation is not only free from the shortcomings of other known methods. It also gives the opportunity (unlike other methods) to take into account the multiplicity of identical estimates of the yield components of the investment portfolio.

The described method also makes it possible to increase the validity of the allocation of resources. In practice, this method has been successfully applied to the evaluation of various options for portfolios in some commercial banks.

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